Generalized Baseline Optimization

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1. Introduction

This memo is an attempt to facilitate a discussion on baseline optimization at Braidwood. The main factors considered in this memo are the risk presented by current uncertainty in Δm^2, and the relationship of rate and shape analyses to systematic error. The most appealing outcome would be to find a baseline that is suitable both as a phase I rate experiment and a phase II shape experiment.

Throughout this memo I will use kinematic phase (defined in Appendix A) as a way to study the optimizations.

2. Optimizing as a Function of Systematic Error

In order to understand the transition from rate dominated sensitivity to shape dominated sensitivity, it is instructive to study sensitivity for rate and rate+shape analyses as a function of the systematic error.

The location of the optimal baseline in the rate analysis is dependent on both the location of the oscillation maximum (kinematic phase = 90º) and the $1/r^2$ statistics fall off. In the systematic limit ($\sigma_{sys} >> \sigma_{stat}$) we expect the optimal baseline to be at the oscillation maximum, because the statistical error is dwarfed by the systematic error making the $1/r^2$ variations immaterial.

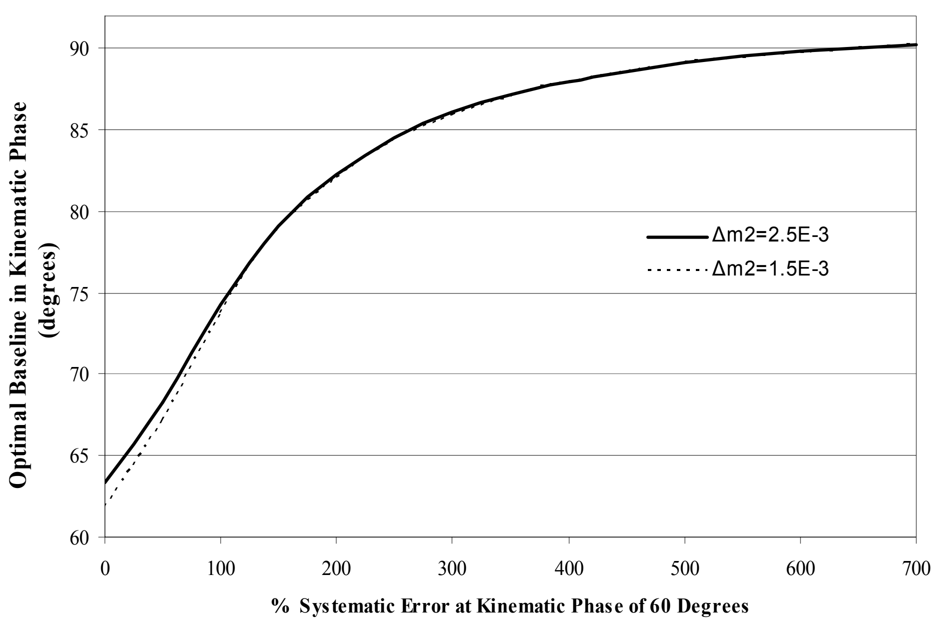

Figure 1: Optimal kinematic phase, for a counting analysis, as a function of the systematic error. The percent systematic error (relative to statistical error) is fixed at a kinematic phase of 60º and is therefore a fixed absolute systematic error.

Table 1: The percent systematic error relative to the statistical error at 60º for a range of baseline and relative normalization error scenarios.

Scenario	Systematic Error
50 tons, 0.6% Rel. Norm. Error	271%
100 tons, 0.6% Rel. Norm. Error	323%
500 tons, 0.6% Rel. Norm. Error	722%
1000 tons, 0.6% Rel. Norm. Error	1021%
50 tons, 0.25% Rel. Norm. Error	113%
100 tons, 0.25% Rel. Norm. Error	135%
500 tons, 0.25% Rel. Norm. Error	301%
1000 tons, 0.25% Rel. Norm. Error	425%

Figure 1 shows the optimal baseline for the counting experiment as a function of the percent systematic error relative to the statistical error measured at a kinematic phase of 60º (the percent systematic error is measured a particular phase so that it correspond to a fixed absolute systematics error). As expected, the figure shows, in the systematic limit, the optimal baseline is essentially 90º[1]. In the statistically limited case, where $\sigma_{sys}=0$, the phase of the optimal baseline is not obviously intuitive. Figure 1 shows the optimal baseline in this limit to be about 63º.

In the rate+shape analysis, the location of the optimal baseline is more complicated. In this case, we expect the optimal baseline to be somewhat away from 90º, because at that location the shape distortion is tightly coupled to the overall normalization error.

Figure 2 shows baseline optimization studies for rate and rate+shape analyses in the statistics limit (left) and the systematics limit (right). In the statistics limit case, the rate+shape sensitivity is very similar to the rate alone sensitivity. The optimum, in this case, is at a slightly smaller phase than for the rate analysis, and the minimum is quite broad. In the case of a systematics limited counting analysis, the optimizations for the rate and the rate+shape analyses are very different. The optimum for the rate+shape is at about 45º with a second local minimum at about 115º. The optimal baseline for a rate analysis is close to a local anti-optimal spot for the rate+shape analysis.

The incompatibility of rate and shape optimizations in the systematic limit forms a powerful argument against locating the experiment at a phase of 90º, because this angle is only optimal for rate in the systematic limit and in that limit the shape information dominates the sensitivity. The baseline Braidwood design, of 1800 meters, is essentially this wrong solution with a phase of 87º for the current best fit value of Δm^2 of 2.4×10^{-3}.

The proper way to optimize for rate alone is to choose the optimal phase for systematic error of about 150% with respect to the statistical error. Beyond this level, the return on increased statistics is diminishing for a rate only analysis. The 150% systematic error optimum is at a phase of 79º or a baseline of 1630 for the best fit Δm^2.

[1] The slight deviation from 90º in the systematic limit is due to the fact that 3.6 MeV is not exactly the average of the neutrino energy spectrum.

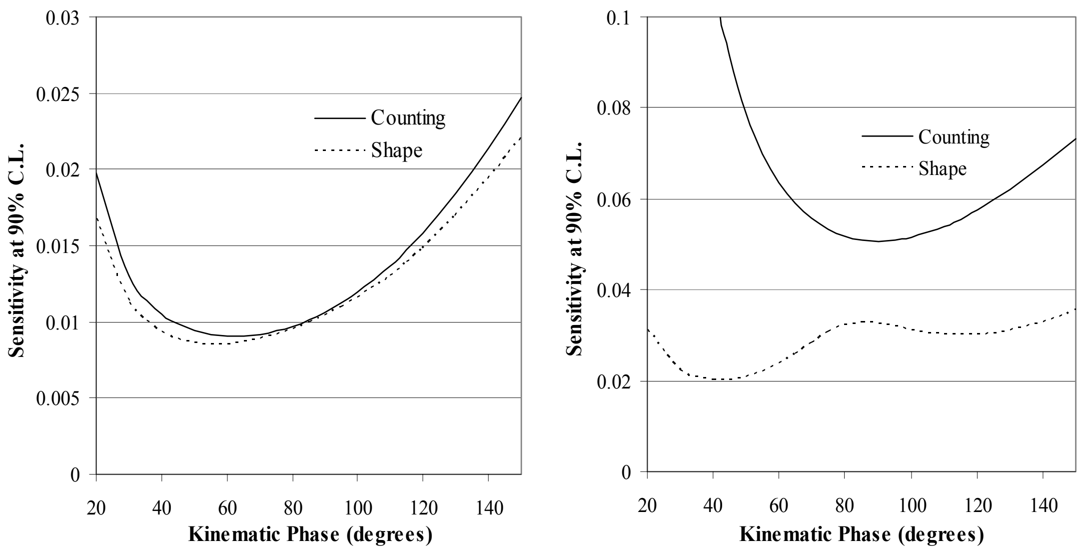

Figure 2: Shows a comparison of baseline optimization studies for counting and rate+shape with no systematic error (left) and a systematically limited counting analysis (right), with 6 times the systematic error compared to statistical error as measured at a phase of 60º.

3. Simultaneously Optimizing for Shape and Counting

Figure 3 combines the optimization studies for rate and rate+shape analyses with 150% systematic errors and a systematics limited shape analysis. In the systematics limit the shape optimum is at a phase of about 45º while the optimum for a rate only analysis is 79º. The rate optimum is quite wide. When you consider the effects of adding the shape information at 150% systematic error, the optimum both shifts to lower kinematic phase and widens. Figure 3 suggests a compromise baseline range of 55º to 60º which would allow for an essentially optimal reach in both a rate-dominated phase I and a shape-dominated phase II.

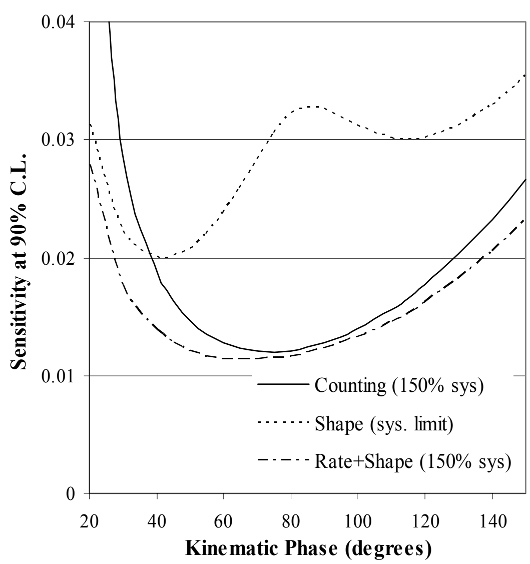

Figure 3: Baseline optimization studies for the optimal rate analysis (150% systematic error) and for the systematics limited shape analysis.

4. Risk Associated with Δm^2

Table 2 lists kinematics phases for several possible values of Δm^2 and far detector baseline. If we consider the rate optimum (79º) and shape optimum (45º) as the upper and lower bounds of acceptable baselines and give a special preference to baselines in the compromise region (55º to 60º), then the optimal baseline should be somewhere between 1250 and 1500 meters.

Table 2: The value of the kinematic phase for various choices of Δm^2 and far detector baseline.

Δm^2	Baseline			
	1000 m	1250 m	1500 m	1750 m
3.0×10^{-3}	60.6º	74.8º	91.0º	106.1º
2.5×10^{-3}	50.5º	63.2º	75.8º	88.4º
2.0×10^{-3}	40.4º	50.5º	60.6º	70.7º
1.5×10^{-3}	30.3º	37.9º	45.5º	53.1º

5. Discussion

Within the optimal region of 1250 to 1500 meters, selecting an exact baseline depends on how you handicap factors such as the uncertainty in Δm^2, the relative importance of rate and shape information, and the importance of background. For example, if you believe

1. Δm^2 will increase or
2. shape information will be more important rate

then you may prefer a baseline closer to 1250 meters. On the other hand, if you believe

1. Δm^2 will decrease or
2. rate will be more important then shape

then you may prefer a baseline closer to 1500 meters.

The importance of background sources, like ^9Li, which puts a peaked structure in the neutrino energy distribution, may also affect the choice of baseline. These backgrounds were not considered in the rate+shape optimization studies presented in this memo. They could make a shape analysis much less sensitive, by simulating or masking a shape deformation. If you do not believe that such background will be significant then you might believe that a strong case for a shape analysis can be made. This suggests moving towards the shorter baseline. But, if you believe that ^9Li will be significant you might also prefer a shorter baseline (at least for a flat overburden site like Braidwood), because it would boost the neutrino the statistics by $1/r^2$ while keeping the background rate constant.

Appendix A: Definition of Kinematic Phase

It is desirable to do a baseline optimization study in a way that is independent of a particular value of Δm^2. This can be achieved by optimizing the kinematic phase. Kinematic phase is defined to be the argument of the oscillatory factor in the neutrino oscillation probability formula

$$P(\overline{\nu}_e \rightarrow \overline{\nu}_x) = \sin^2 2\theta_{13} \sin^2(\underline{1.27 \times \Delta m_{13}^2 L / E})$$

where E_ν is replaced with the average observable neutrino energy (=3.6 MeV). For convenience the phase is converted into degrees. So

$$KinimaticPhase \equiv 1.27 \times \Delta m^2 \frac{L}{3.6 MeV} \times \frac{180}{\pi}.$$

Figure A1 shows baseline scans for two different values of Δm^2, as a function of kinematic phase. While the absolute value of the sensitivity depends on the choice of Δm^2 — primarily because a larger Δm^2 implies a closer detector (higher statistics) for the same phase — the baseline optimization is independent of the choice of Δm^2. Table 2 gives the kinematic phase for several values of Δm^2 and baseline.

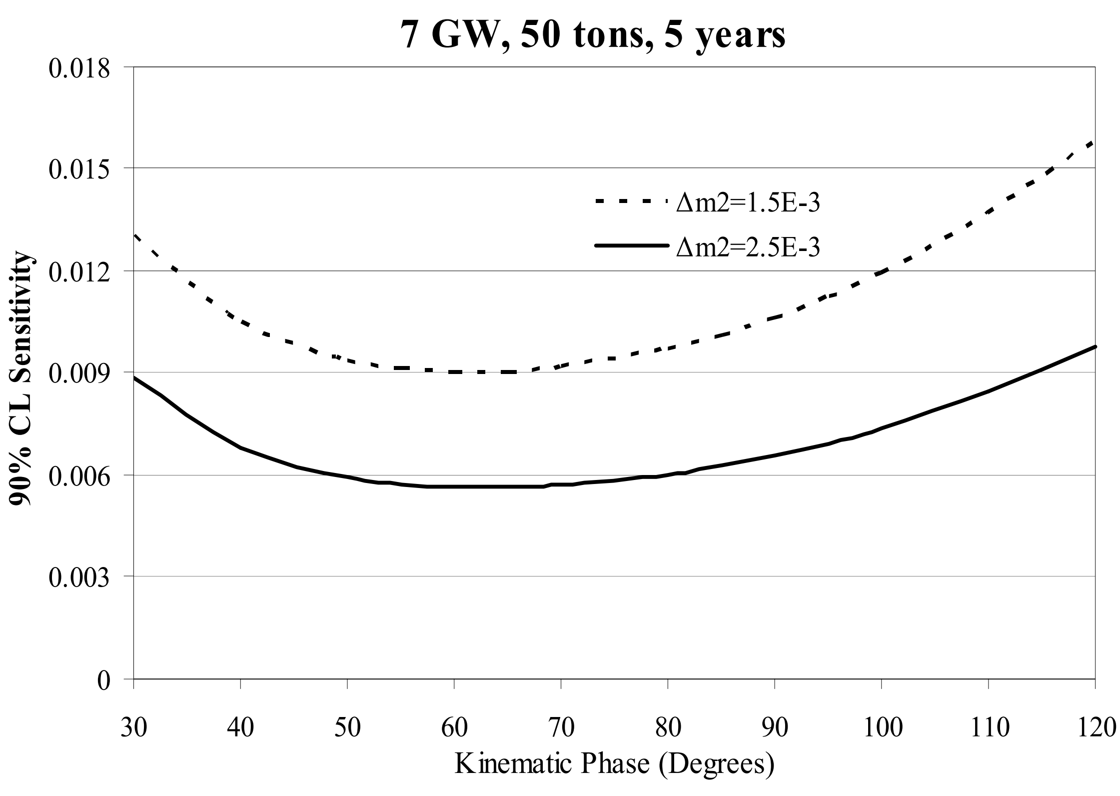

Figure A1: This figure illustrates that by using the kinematic phase optimization of baseline can be studied independent of a specific Δm^2.