Idea for a 3-step Method for Constraining the Energy Scale and Offset Error

Janet Conrad, May 11, 2005

This memo outlines a 3 step method which could be used to constrain the relative energy scale and offset between IBD and ES events to the level required for the ES analysis. Figures 1 and 2 show how a relative energy scale and offset translate into an error on the total ES event rate. From these plots it is clear that we must aim for very small relative errors. The purpose of this memo is to show a proof of principle that the required levels can be obtained, although there are still practical questions to be addressed and improvements to be made to the analysis.

The steps to constraining these errors are:

- Fit the IBD events for both scale and offset. This will be the scale and offset associated with e^+ in the detector
- Fit the muon decay events for a relative energy offset of the e^- compared to the e^+. This fit allows separate normalization for decays to e^+ and e^- and is insensitive to a scale shift.
- Having fixed the relative offset, fit the β^- decays in the detector to obtain the relative scale error.

1 Throwing and Fitting the Distributions

This analysis uses four distributions, shown in figure 3. They are the IBD event distribution (top left), the michel distribution from μ decay (top right), the ^{6}He β decay distribution (bottom left) and the ^{12}B distribution (bottom right). The distributions are thrown randomly in order to mimic the statistical fluctuations. The distributions can be thrown with or without an energy scale correction and energy offset.

The program uses Minuit to find the best fit, varying the scale factor and/or offset and the normalization. The variance between the thrown distribution and the fit distribution is minimized. Because of the odd effect in the β parameterization near zero, the fit for β events is for 0.5 MeV and higher. In the future, we may want to use this cut on IBD and muon decay fits also. This should not change the conclusion of this study.

The energy smearing is included, but is probably underestimated. All of the thrown distributions have $10\% \times \sqrt{E}$ gaussian smearing. This is illustrated by figure 4, which shows smeared energy as a function of true energy for the ^{12}B events. This is smaller than the 250 keV smearing at 3 MeV estimated by Lev Mikaelyan, but is consistent with what Braidwood is claiming. The smearing is also underestimated because the event is always assumed to have the bin center energy value before the smearing is applied. In the future, the distribution needs to be properly spread across the bin and then smeared.

In principle, smearing should not pull the fit if it is well understood and included in the fit via a smearing matrix. This will be a future step as this study develops. For the results presented here, while the thrown data does include smearing, the fit does not include smearing.

In order to get the error on the procedure, a large number of experiments (> 100) are thrown. The fit parameters are plotted and are fit to Gaussians. The mean of the Gaussian should shift from zero when the events are thrown with a scale factor, an offset, or both. But the width of the distribution, should not change. The width of the distribution is the measurement of how well we can know the parameter.

2 IBD Events

The IBD events use the flux distribution from the Vogel and Gratta RMP article and the cross section from Vogel and Beacom. The data are fit assuming a 1% error in the reactor flux. This systematic error is added to the denominator when the variance is calculated.

Five million IBD events are thrown in each experiment. Four cases are considered:

- zero underlying scale and offset in the thrown distribution
- scale factor of 0.005 and zero offset in the thrown distribution
- zero scale factor and 10 keV offset in the thrown distribution
- scale factor of 0.003 and 5 keV offset in the thrown distribution

The distributions are then fit allowing for a scale factor and offset.

Figures 5 and 6 shows that the extracted parameters are an excellent representation of the underlying scale and offset. Note that while the means of the parameters shifted, the widths are consistent for all four cases. Also, the widths of the distributions are very narrow. The scale offset can be determined to 0.0009, which is consistent with the constraint on energy scale from IBDs being found in the oscillation analysis for Braidwood by Mike Shaevitz. The offset can be determined to 0.003 keV.

3 Muon Decay Events

After correcting the energy scale and offset using the IBD events, the next step is to correct for a relative offset in the energy between ES-type and IBD-type events. In muon decay, the michel electron from μ^- decay, at low energy, mimics an ES event and the michel positron at low energy from μ^+ decay mimics an IBD-event.

The michel distribution is well known. The recent results from the TWIST collaboration have shown that this standard model parameterization is good to 0.1%. The michel distribution from μ^+, which produces and e^+, is offset from zero by 1 MeV to simulate the annihilation photons. The michel distributions from μ^+ and μ^- are added together before they are fit.

The relative normalization of μ^+ to μ^- decay events is not well known (mainly due to the atmospheric production rates, but also partly due to the μ^- capture rate) and is left as a parameter. For this study, the μ decay distributions are thrown with equal rates of μ^+ and μ^- using the standard model expectation.

Five million decays were considered in each experiment. This is based on an estimated rate of stopped muons at Braidwood of about 100/ton/day. Thus in 900 days at 70 tons, there are 6 million events.

Lack of knowledge of the relative normalization is what prevents the muon decay sample from constraining the energy scale. In the fit, a scale shift is confused with a change in the normalization, since the spectrum is only fit up to 5 MeV. While in this study there was absolutely no constraint on the normalization in the fit, in the future we could consider applying one based on the level of knowledge of the μ^+ to μ^- ratio, if it is sufficiently well known to be useful.

The distribution is sensitive to energy offsets. This is because the annihilation photons in the μ^+ decays offset the visible energy distribution by 1 MeV, leading to a kink in the total michel distribution. Figure 7 shows three cases of underlying distributions:

- zero underlying scale and offset in the thrown distribution
- zero scale factor and 10 keV offset in the thrown distribution
- scale factor of 0.003 and 5 keV offset in the thrown distribution

(A scale factor of 0.005 and zero offset in the thrown distribution looks identical to zero underlying scale and offset in the thrown distribution, since the distribution is insensitive to the scale.) The means reflect the three offsets in the underlying distributions. The widths are narrow and less than 0.002 keV.

Adding the width of the muon decay offset distribution in quadrature with the width of the IBD offset distribution gives the relative error on the energy offset between ES-like and IBD-like events. This is about 2 keV. This is much smaller than the 10 keV goal.

4 β^- Events

Having determined the relative energy offset, the the distributions for β^- decays in the energy range of interest (near 3 to 5 MeV) can be used to determine the relative energy scale.

For this study, parameterizations meant to mimic ^{6}He and ^{12}B were used, with 37k and 150k events thrown for each experiment, respectively. This is the expected rate for Braidwood. Figure 8, top, shows the distributions for no underlying scale shift and for a 0.005 factor. The means of the distributions lie as expected, given the widths. But the widths of the distributions are very large. They represent a relative error on the energy scale of 0.007 and our goal was 0.005.

If we had ten times the statistics, the distributions tighten up considerably. This is shown in figure 8, bottom. Again, the means of the distributions lie as expected, The widths are 0.002. You would expect that with 10 times the statistics that the distribution would be a factor of 3 narrower, and it is. Adding this in quadrature with the IBD scale error leads to a 0.0025 error on the relative scale, substantially smaller than the goal. A factor of four more statistics would yield a 0.0035 scale error for ES-like events, which when added in quadrature would be acceptable. Therefore, this study indicates that we are about a factor of four in statistics short at Braidwood when one only considers ^{6}He and ^{12}B decays.

5 Conclusions

This memo is meant as a proof of principle. We now need a practical method to carry out the plan. The main flaw is the lack of a strong constraint on the energy scale of the ES-like events. This could be addressed in a number of ways:

- If the relative normalization of the muon decay electron and positron michels is sufficiently well known, then this distribution can help constrain the scale error as well as the offset.
- There are other taggable β^- spallation products. We may also be able to use identifiable contaminants (perhaps ^{40}K?)
- We may be able to identify a β^- decaying gas which could be bubbled through the detector. This element would need to be a pure β^- decay (or have only a low rate of decays to excited states) which is easily producible at an accelerator (that is, Fermilab) It would need to be sufficiently long lived to be transportable to the detector, but decay quickly enough that we could come back on line after a short period. And it needs to be non-reactive.

Ideas are welcome!

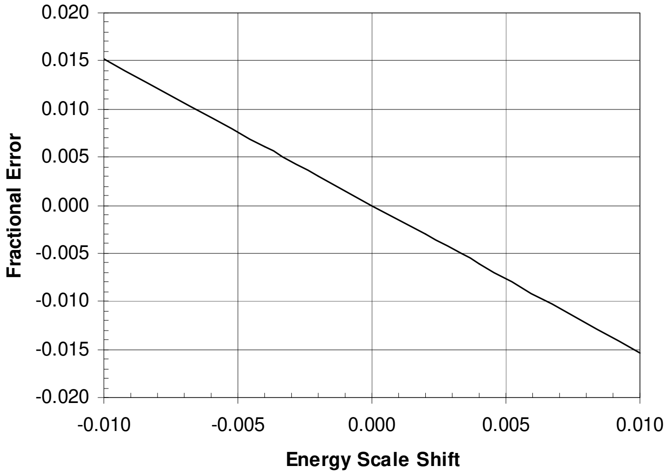

Figure 1: Effect of a relative energy scale factor between ES and IBD events, on the error on the total ES events in the analysis. To put this in perspective: we are aiming for an 1% measure on the total ES events.

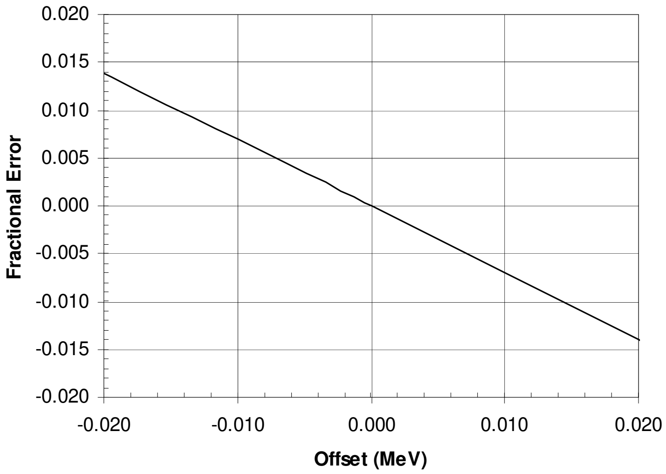

Figure 2: Effect of a relative energy offset on the error.

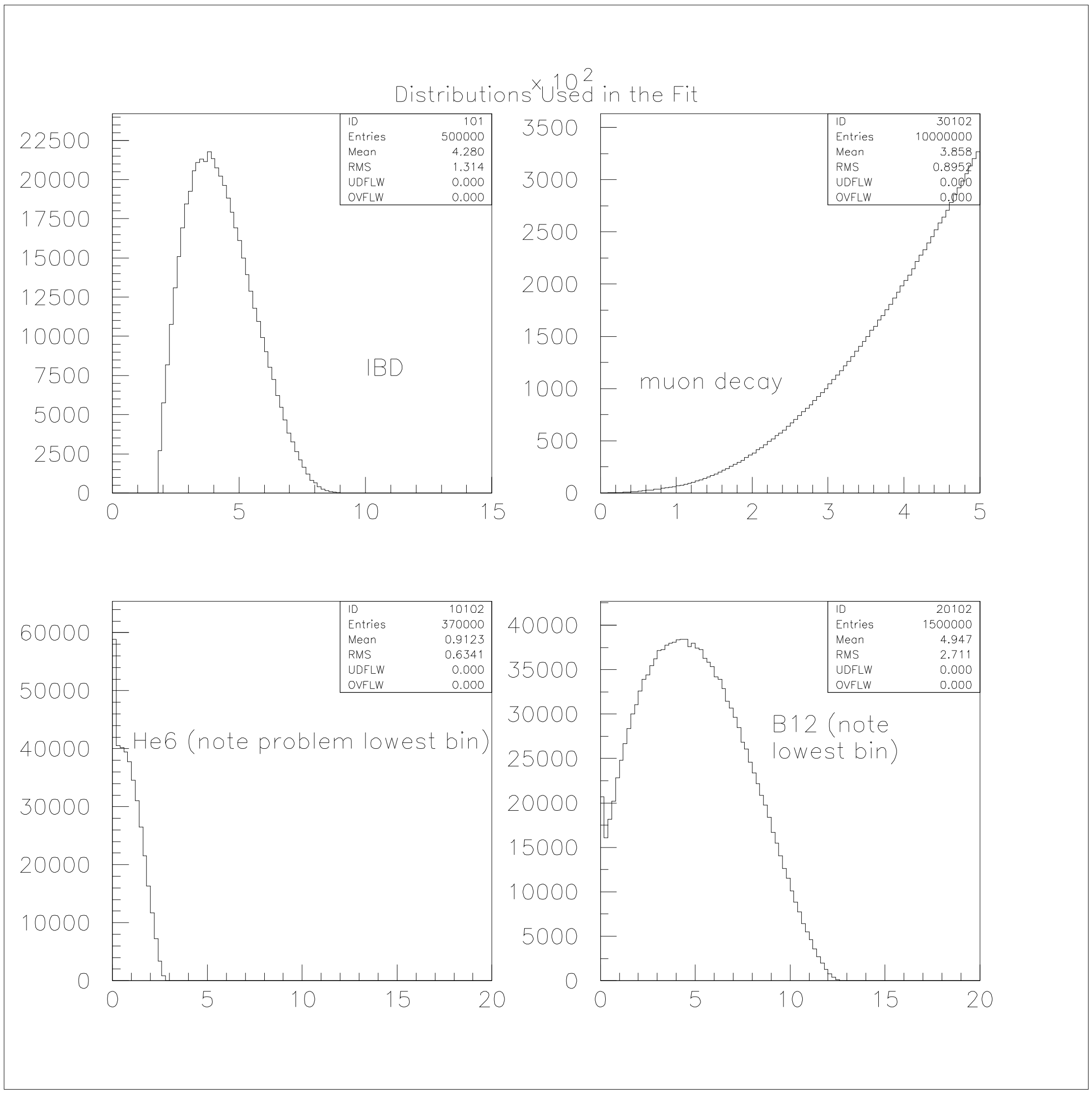

Figure 3: Distributions used in this study. Because the β^- functions fail as the energy goes to zero, fits are done for 0.5 MeV and up.

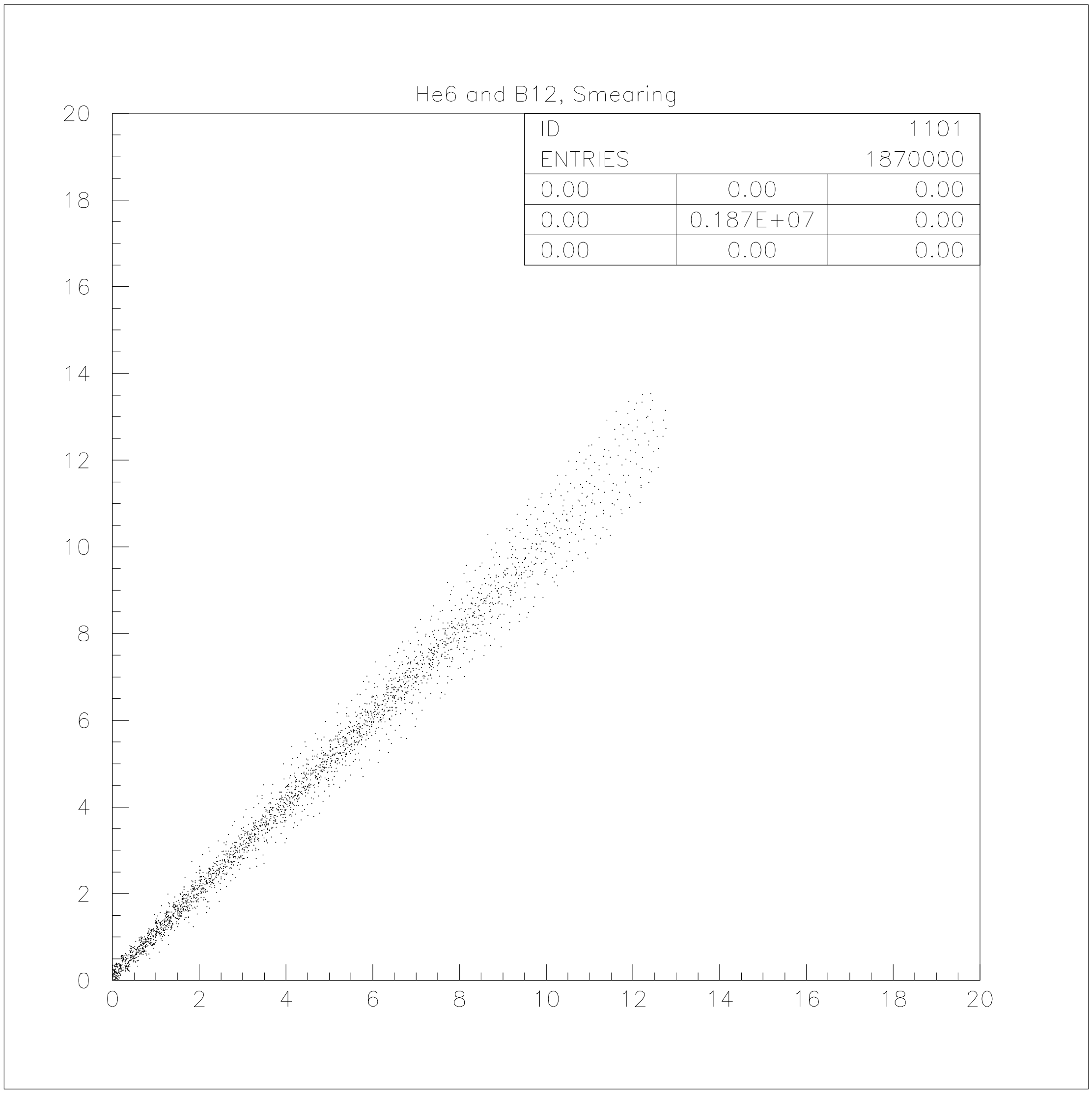

Figure 4: Example: Smearing for ^{12}B events.

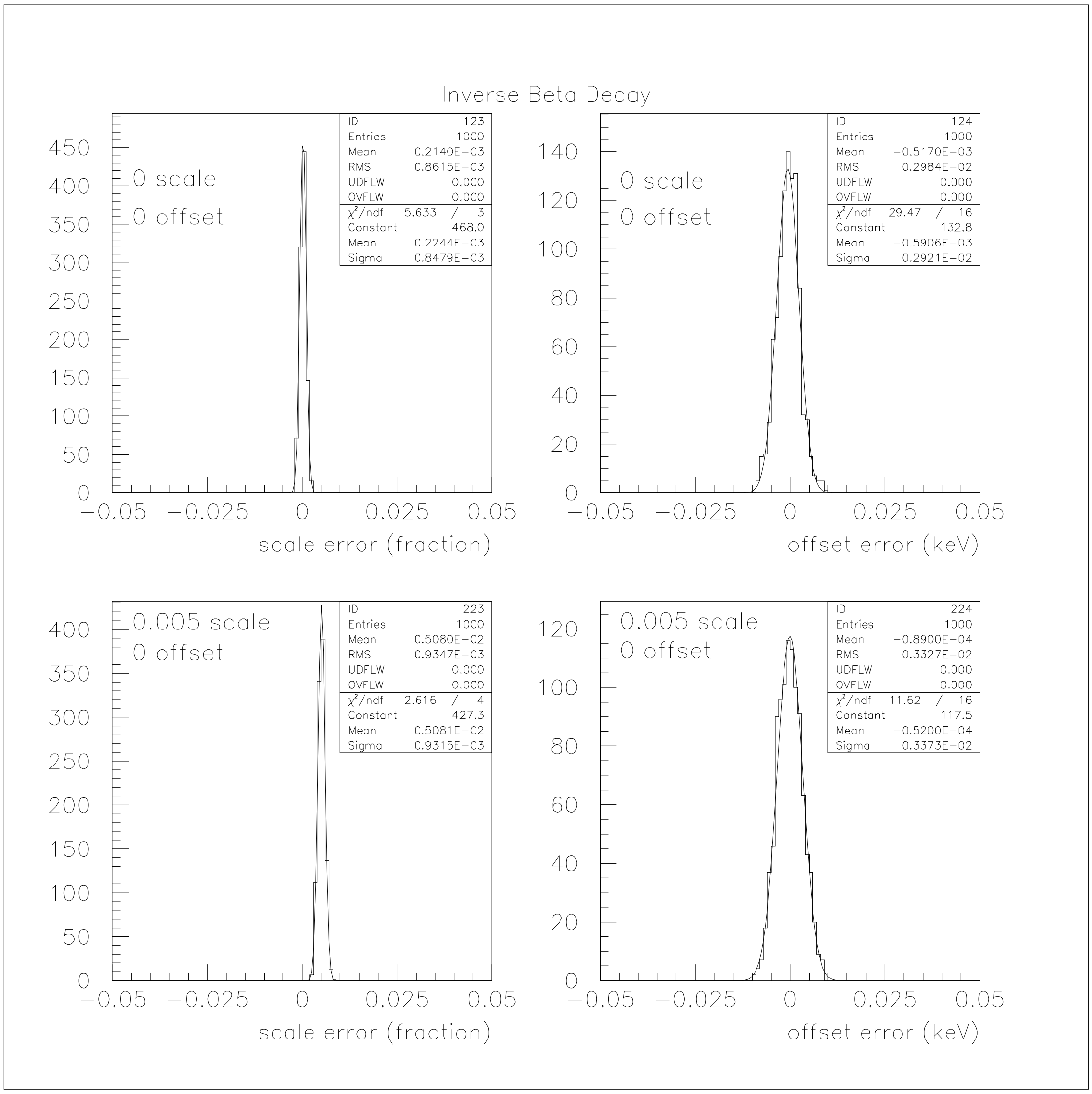

Figure 5: Parameters found for two underlying energy scales and offsets in the IBD events

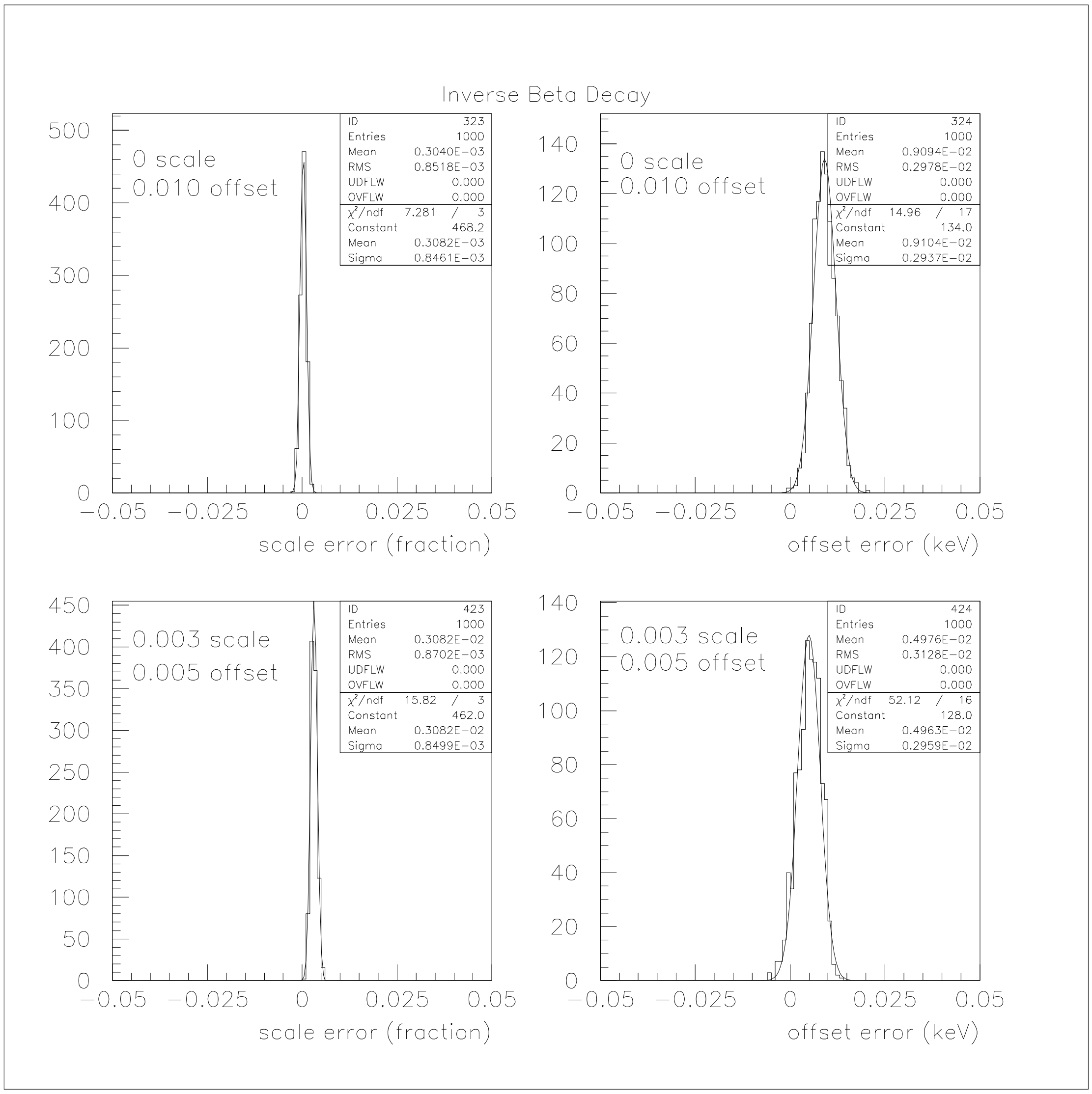

Figure 6: Parameters found for two underlying energy scales and offsets in the IBD events

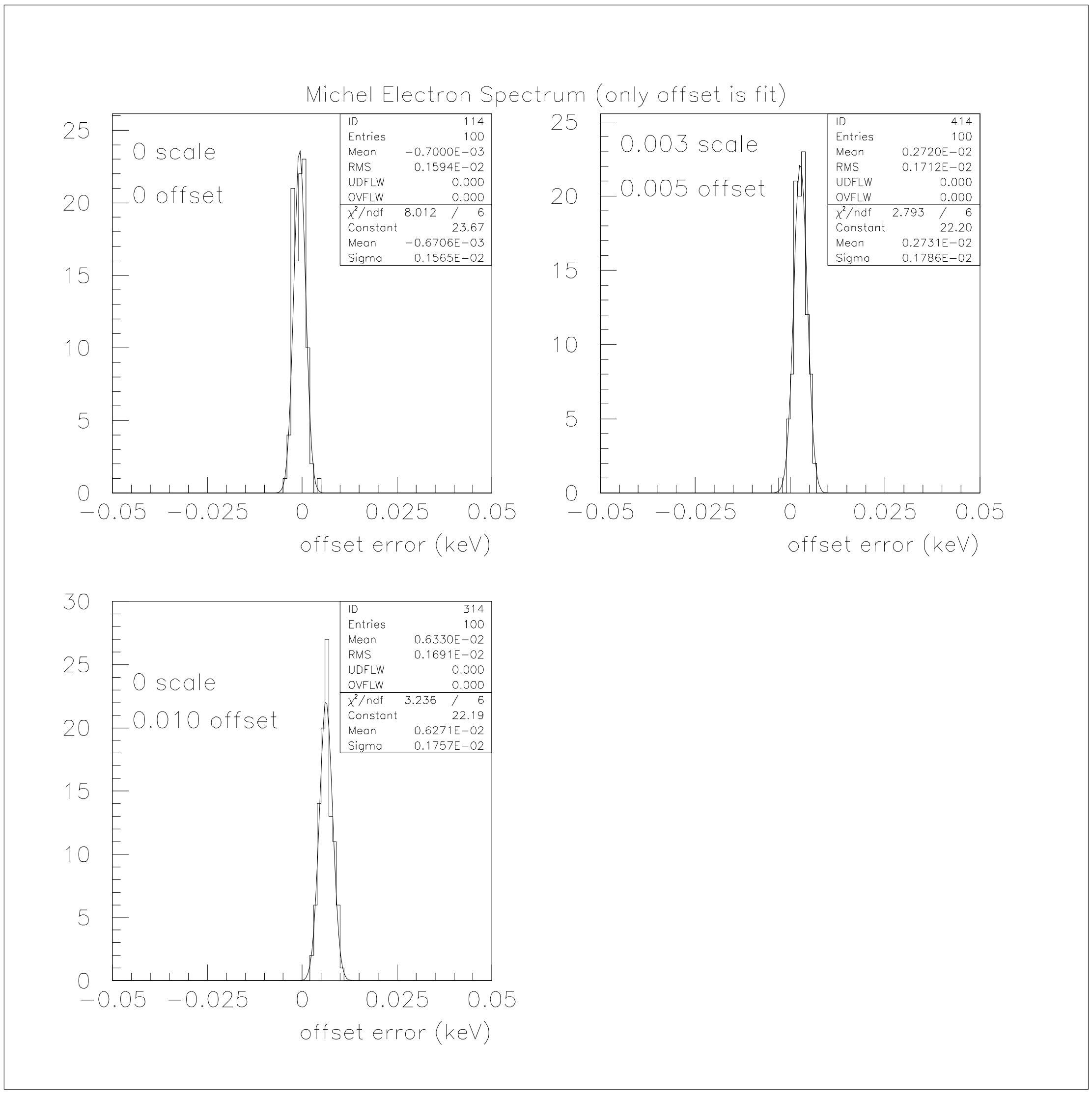

Figure 7: Parameters found for underlying relative offsets between michel electrons and michel positrons.

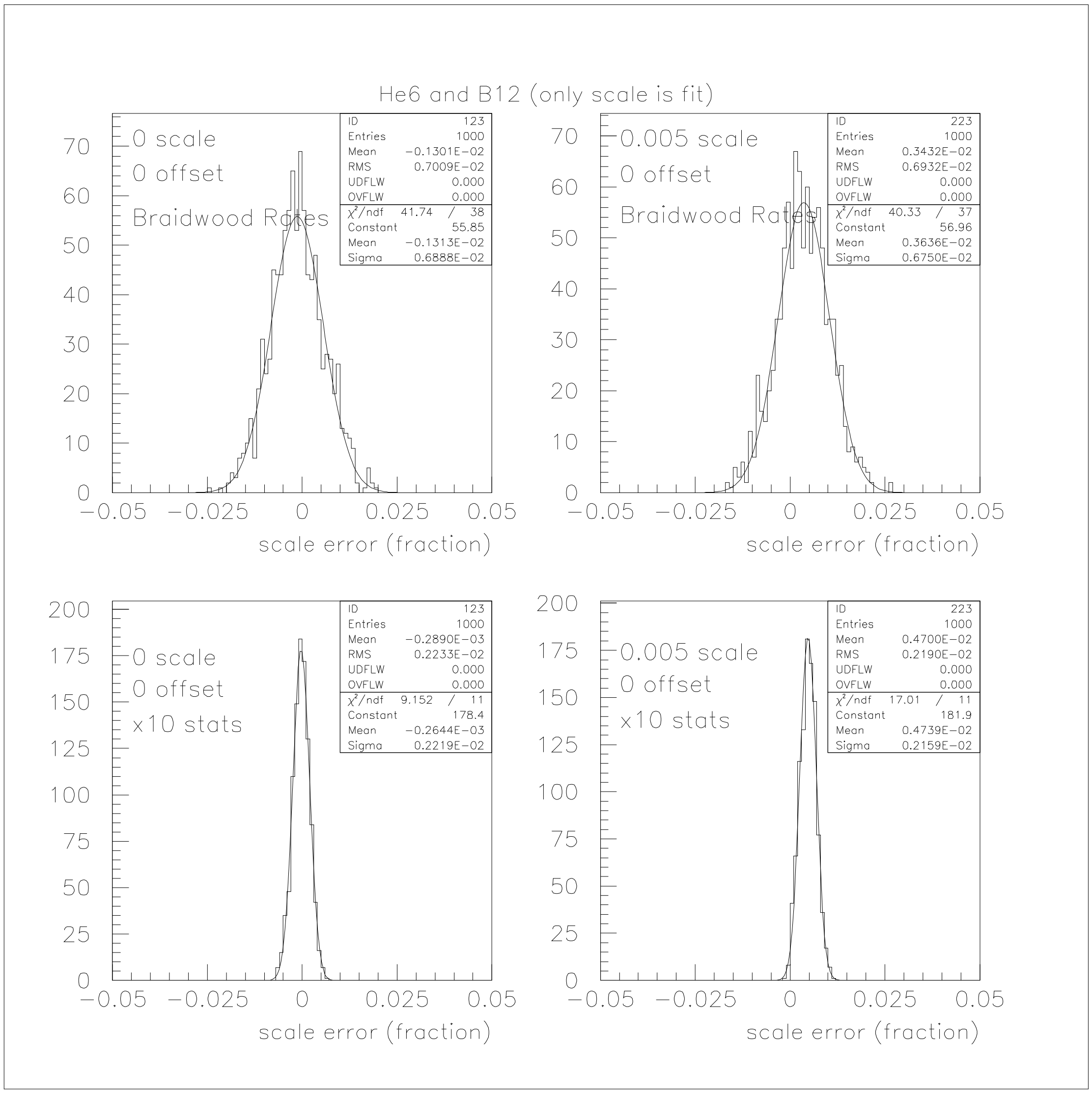

Figure 8: Parameters found for underlying scale factors between IBD events and β^- decay events. Top: Braidwood statistics; Bottom: $\times 10$ Braidwood statistics.